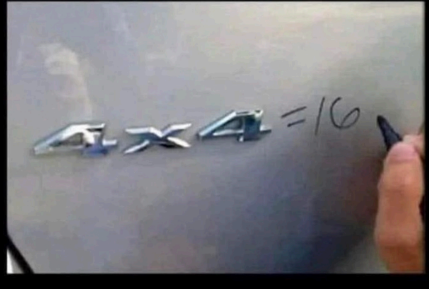# Basic Multiplication

This poor guy was driving around with this unsolved problem on his truck, had to fix it for him...

4x4 =16

Japanese family found calculator in their son's room

There are many ways to multiply:

Way 1: Area Model/Grid/Box Method – This method shows clearly what is happening and is is great for understanding, especially for those who prefer a visual understanding as it can be linked to finding the area of rectangles. It also comes in handy in other areas as it is a relatively natural method and can be used to help with expanding quadratics and multiplying polynomials.

Ways 2 and 3: Column Method – Way 3 is very widespread and more likely to be understood by parents and grandparents. It is also a nice algorithmic method that allows space to understand what is going on.

Way 4: The Lattice Method (Napier's Bones/Gelosia Method) – This is great if your main goal is just to get multiplication done, however doesn't do anything to aid understanding. The area model leads to this method. Weaker students like this method as a student who doesn't understand what multiplication is about might be able to reproduce this method and get the answer right every time. The problem is that this take time to set up and does not advance any mathematical concepts (it destroys place value).

Way 5: Criss Cross Method – This is not a very natural method, but it is quick and works for multiplying any n by n multiplication problem.

Way 6: Chinese Stick Multiplication (Line Method/Japanese Multiplication) – This method helps students to think more about what the multiplication of certain digits is providing to the product. Such as the multiplication of a ones digit and another ones digit will provide the ones digit of the product. It's one thing to know how to carry out a procedure (like long multiplication), but this is only useful when a student knows why that method works!

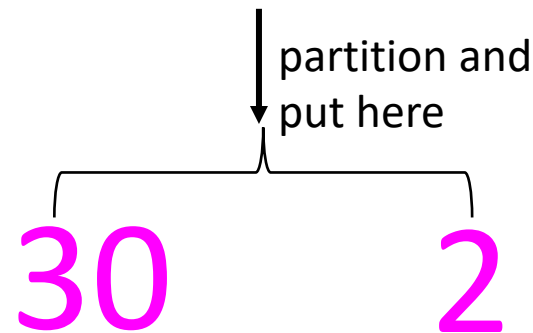Note: We will look at the Criss Cross Method and Chinese Stick Multiplication method separately at the end
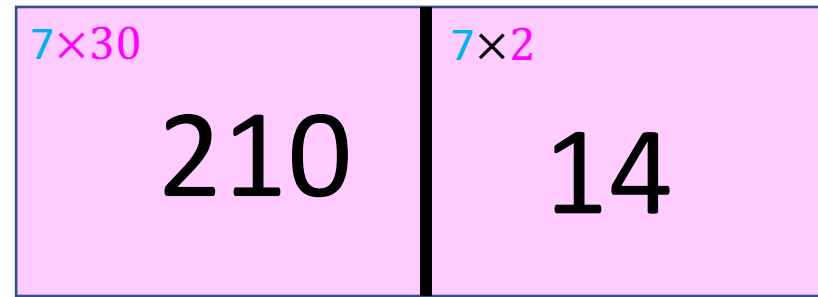
Example 1

$$32 \times 7$$

Way 1   **Area Model/Box/Grid Method**

Split/partition the numbers up into their place values

32 = 3 tens (30) and 2 ones (2) which means 30 +2

partition and put here

30   2

7 = 7 ones (7)   put here   7
which means 7

7×30	7×2
210	14

Method:

For each box we FIRST multiply the number on the top of the box with the number on the left of of the box.

We then add all the numbers in the boxes together.

add all numbers in the boxes together: $210 + 14 = 224$

Way 2

3 2

7

Note: we write 30 and not 3
since 3 is in the tens place

2×7 = 14

30×7 = 210

$210 + 14 = 24$

Way 3 Long Multiplication (this is just an algorithmic way to do way 1)

② do this next   ① start here

$$3\ 2$$
$$\quad 7$$
x

$3{\times}7$ $7{\times}2$

add the **1** after from previous step

$3{\times}7 + 1 = 22$

$7 \times 2 =$**14** we carry the **1** from the 14 up like we did with addition/subtraction

$\Rightarrow$

② do this next   ① start here
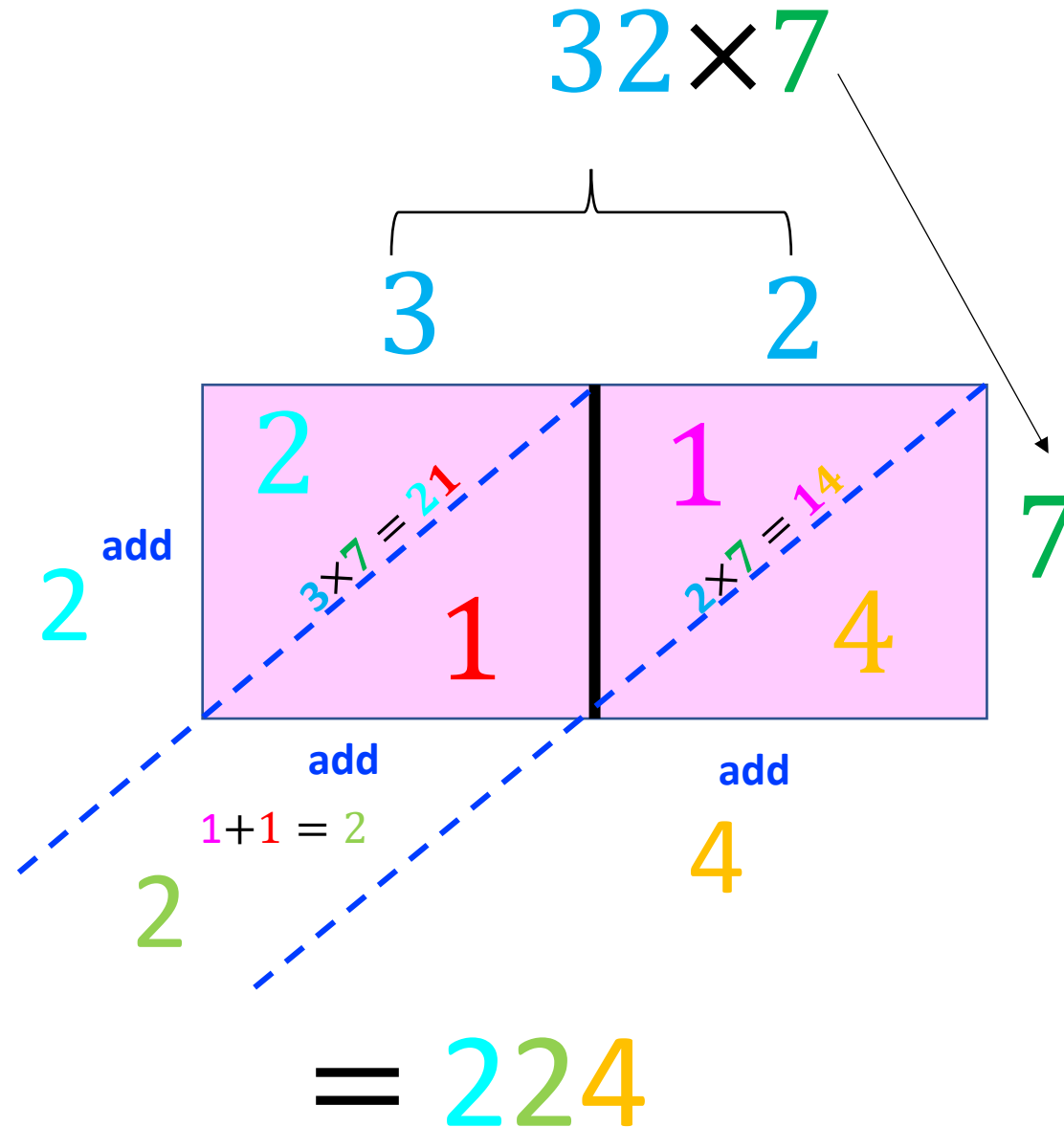
1
$$3\ 2$$
$$\quad 7$$
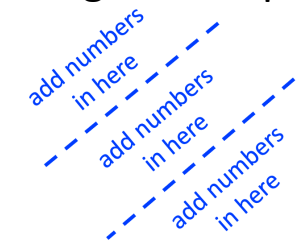x

$$2\ 2\ 4$$

$3{\times}7 + 1 = 22$

Way 4

Lattice Method/Napier's Bones/Gelosia

Method:

Step 1:
For each box we FIRST multiply the numbers on the top of the box with the number to the far right of the box (7) and THEN split the digits of the number you get from multiplying (this number is shown on top of the diagonal) across the dashed diagonal that divides each box.

Step 2:
Add the numbers in each of the separate diagonal strips

add numbers in here
add numbers in here
add numbers in here

(start on the right). These numbers form our answer (from left to right).

32×7

3   2

2   1

3×7 = 21   2×7 = 14

1   4

7

add

2

add   **add**

$1+1 = 2$

2   4

$= 224$

Example 2

$$43 \times 82$$

Way 1

Area Model/Box/Grid Method

Split/partition the numbers up into their place values

43= 4 tens (40) and 3 ones (3) which means 40 +3

partition here

	40	3
80	80×40 **3,200**	80×3 **240**
2	2×40 **80**	2×3 **6**

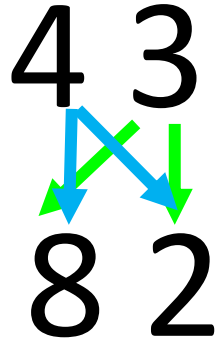82 = 8 tens (80) and 2 ones (2) which means 80+2

partition here

Method:

For each box we FIRST multiply the number on the top of the box with the number on the left of of the box.

We then add all the numbers in the boxes together.

add all numbers in the boxes together: 3,200+ 240 + 80 +6 = 3,526

Way 2

4 3

8 2

3×2 = 6

3×80 = 240

Note: we write 40 and not 4 since 4 is in the tens place

40×2 = 80

Note: we write 80 and not 8 since 8 is in the tens place

40×80 = 3,200

Method:

Multiply each of the colour pairs and then add the results

$$6 + 240 + 80 + 3,200 = 3,526$$

Way 3 — Long Multiplication (this is just an algorithmic way to do way 1/2

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Step 1
do every multiplication with the pink numbers
(carry if we have a two-digit number, just like with addition)

we write our answers on the top line

$$\begin{array}{r} 4\ 3 \\ \times\quad 8\ 2 \\ \hline 2\times4 \ \ 2\times3 \end{array}$$

next ⇒

Step 2
do every multiplication with the blue numbers
(carry if we have a two-digit number, just like with addition)

we write our answers on the bottom line

$$\begin{array}{r} 4\ 3 \\ \times\quad 8\ 2 \\ \hline \end{array}$$

done in step 1 → 2×4 2×3

8×4 8×3 0

+2

This is 24. We need to carry the 2 over since 2 digits

always put a zero in this place

Step 3
next ⇒

Do each calculation and then add

$$\begin{array}{r} 4\ 3 \\ \times\quad 8\ 2 \\ \hline 8\ 6 \\ +\ 3\ 4\ 4\ 0 \\ \hline 3\ 5\ 2\ 6 \end{array}$$

Note: This example has shown the steps, but you should be able to do just do the 3rd column once you understand the steps

Without all the colour coding this looks like

do every multiplication with the pink numbers
(carry if we have a two-digit number, just like with addition)

$$\begin{array}{r} 4\,3 \\ \times \quad 8\,2 \\ \hline 8\,6 \\ \hline \end{array}$$

next ⟹

do every multiplication with the blue numbers
(carry if we have a two-digit number, just like with addition)

$$\begin{array}{r} {}^{2}4\,3 \\ \times \quad 8\,2 \\ \hline 8\,6 \\ 3\,4\,4\,0 \\ \hline \end{array}$$

always put a zero here

add ⟹

$$\begin{array}{r} 4\,3 \\ \times \quad 8\,2 \\ \hline 8\,6 \\ +\ 3\,4\,4\,0 \\ \hline 3\,5\,2\,6 \end{array}$$
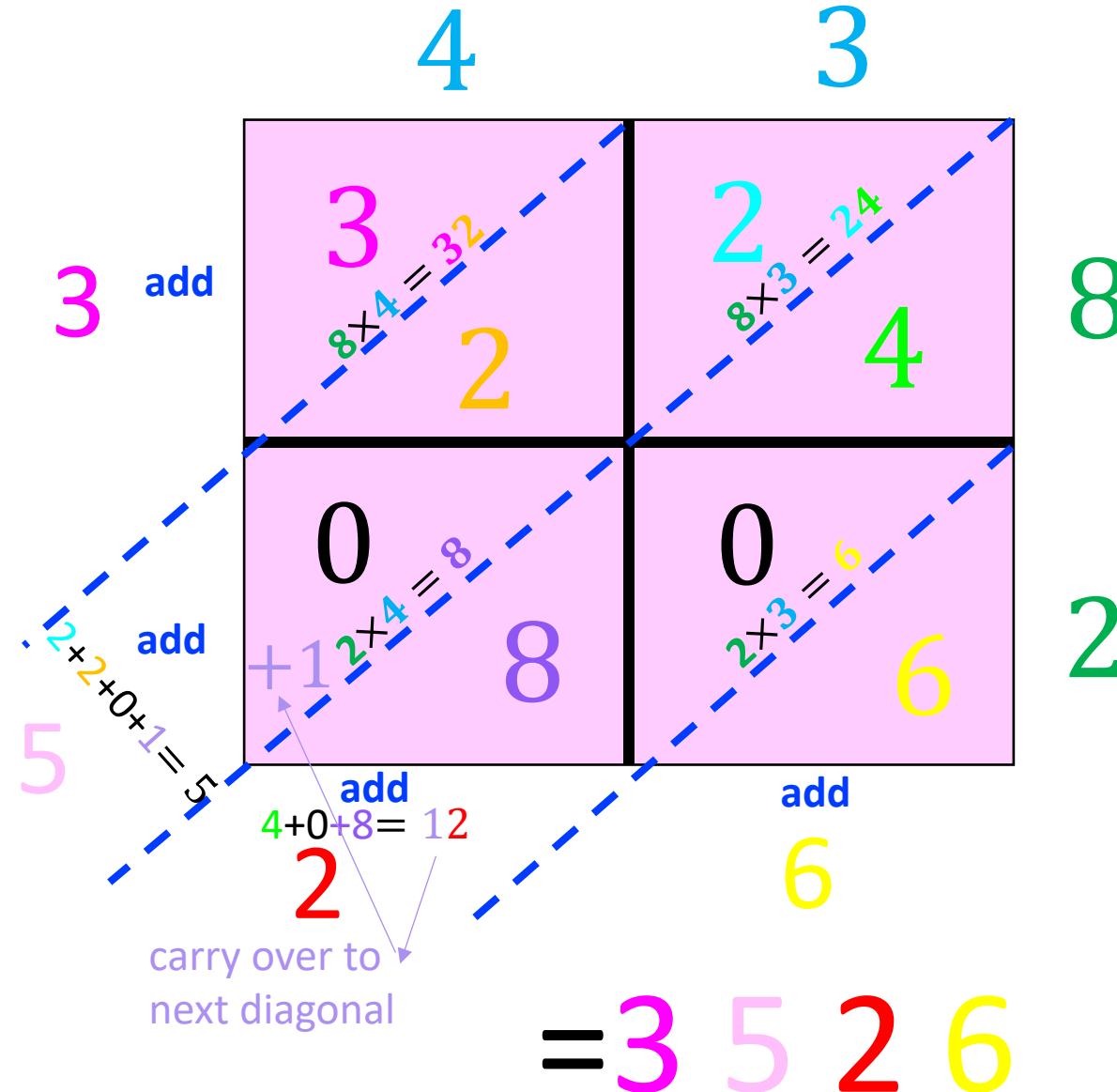
Note: This example has shown the steps to explain, but you should be able to do just do the 3rd column once you understand the steps
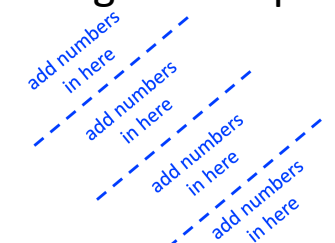
Way 4

Lattice Method

43×82

Method:

Step 1:
For each box we FIRST multiply the numbers on the top of the box with the numbers to the far right of the box and THEN split the digits of the number you get from multiplying (this number is shown on top of the diagonal) across the dashed diagonal that divides each box.

Step 2:
Add the numbers in each of the separate diagonal strips

add numbers in here
add numbers in here
add numbers in here
add numbers in here
add numbers in here

(start on the right). These numbers form our answer (from left to right).

4   3

3 **add**
3      $8 \times 4 = 32$ 2   2 $8 \times 3 = 24$ 4   8

add
0      $2 \times 4 = 8$ +1   8   0 $2 \times 3 = 6$ 6   2

$2+2+0+1=5$
5

add
$4+0+8=12$
2

add
6

carry over to next diagonal

$= 3\ 5\ 2\ 6$

Example 3 612×24

Way 1   **Area Model/Box/Grid Method**

612 = 6 hundreds (600), 1 tens (10) and 2 ones (2) which means $600+10+2$

partition and put here

	600	**10**	**2**
20	20×600 **12,000**	20×10 **200**	20×2 **40**
4	4×600 **2,400**	4×10 **40**	4×2 **8**

24 = 2 tens (20) and 4 ones (4) which means $20+4$
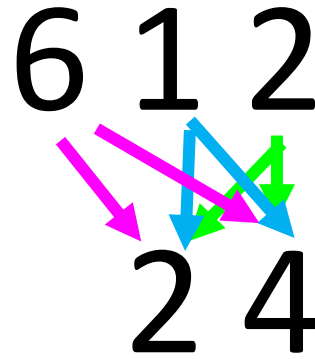
partition and put here

Method

For each box we FIRST multiply the number on the top of the box with the number on the left of of the box.

We then add all the numbers in the boxes together.

$$12,000+200+40+2,400+40+8=14,688$$

Way 2

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6 1 2

2 4

| 2 × 4 | = 8 |

| 2 × 20 | = 40 |

| 10 × 4 | = 40 |

| 10 × 20 | = 200 |

| 600 × 4 | = 2,400 |

| 600 × 20 | = 12,000 |

Method:

Multiply each of the colour pairs and then add the results

$8 + 40 + 40 + 200 + 2,400 + 12,000 = 14,688$

Way 3 **Long Multiplication (this is just an algorithmic way to do way 1/2**

Step 1

do every multiplication with the pink numbers
(carry if we have a two-digit number, just like with addition)

we write our answers on the top line

$$6\ 1\ 2$$
$$\times\ \ \ 2\ 4$$

4×6   4×1   4×2

next ⟹

Step 2

do every multiplication with the blue numbers
(carry if we have a two-digit number, just like with addition)

we write our answers on the bottom line

$$6\ 1\ 2$$
$$\times\ \ \ 2\ 4$$

4×6   4×1   4×2

2×6   2×1   2×2   0

always put a zero here

next ⟹

Step 3

Do each calculation and then add

$$6\ 1\ 2$$
$$\times\ \ \ 2\ 4$$
$$2\ 4\ 4\ 8$$
$$+\ 1\ 2\ 2\ 4\ 0$$
$$1\ 4\ 6\ 8\ 8$$

do every multiplication with the pink numbers
(carry if we have a two-digit number, just like with addition)

$$
\begin{array}{r}
6\ 1\ 2 \\
\times\ \ \ 2\ 4 \\
\hline
2\ 4\ 4\ 8 \\
\hline
\end{array}
$$

next $\Rightarrow$

do every multiplication with the blue numbers
(carry if we have a two-digit number, just like with addition)

$$
\begin{array}{r}
6\ 1\ 2 \\
\times\ \ \ 2\ 4 \\
\hline
2\ 4\ 4\ 8 \\
1\ 2\ 2\ 4\ 0 \\
\hline
\end{array}
$$

always put a zero here

add $\Rightarrow$

$$
\begin{array}{r}
6\ 1\ 2 \\
\times\ \ \ 2\ 4 \\
\hline
2\ 4\ 4\ 8 \\
+\ 1\ 2\ 2\ 4\ 0 \\
\hline
1\ 4\ 6\ 8\ 8 \\
\end{array}
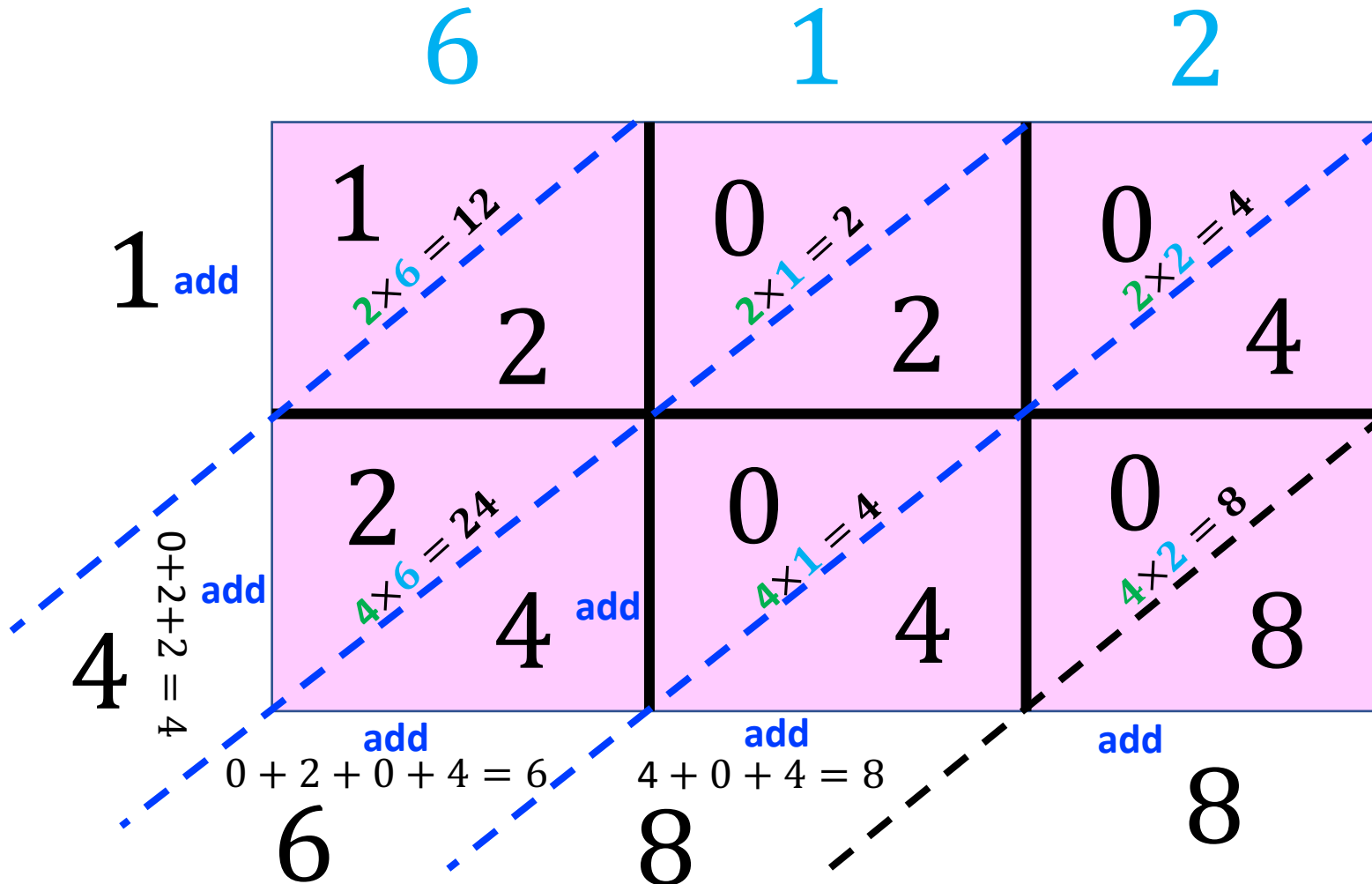$$

This example has shown the steps to explain, but you should be able to do just do the 3rd column once you understand the steps

Way 4

Lattice Method

612×24

Method:

Step 1:
For each box we FIRST multiply the numbers on the top of the box with the number on the far right of the box and THEN split the digits of the number you get from multiplying (shown on top of the diagonal) across the dashed diagonal that cuts up each box.

Step 2:
Add the numbers in each of the diagonal strips (start on the right). These numbers form our answer (from left to right).

Let's do another example, but this this time only using the most common method which is long multiplication way. This example is the same as above, except we need to carry more.

Example 4

$$828 \times 35$$

$$\begin{array}{r} 828 \\ \times\ 35 \\ \hline \end{array}$$

5×8 add the 1

5×2 add the 4 carry the 1 over (since this is 14(

5×8 carry the 4 over (since this is 40)

next $\Rightarrow$

$$\begin{array}{r} 828 \\ \times\ 35 \\ \hline \end{array}$$

5×8 add the 1

5×2 add the 4 carry the 1 over

5×8 carry the 4 over

3×8 3×2 3×8 0
        +2   carry the 2 over
              since this is 24

always put a zero here

add $\Rightarrow$

$$\begin{array}{r} 828 \\ \times\ 35 \\ \hline 4140 \\ +\ 24840 \\ \hline 28980 \end{array}$$

This example has shown the steps to explain, but you should be able to do just do the 3rd column once you understand the steps

Without all the extra colour coding this looks like:

do every multiplication with
the pink numbers
(carry if we have a two-digit
number, just like with addition)

do every multiplication with
the blue numbers
(carry if we have a two-digit
number, just like with addition)

$$
\begin{array}{r}
^{1}8\ ^{4}2\ 8 \\
\times\quad 3\ 5 \\
\hline
4\ 1\ 4\ 0
\end{array}
$$

next $\Rightarrow$

$$
\begin{array}{r}
8\ ^{2}2\ 8 \\
\times\quad 3\ 5 \\
\hline
4\ 1\ 4\ 0 \\
2\ 4\ 8\ 4\ 0
\end{array}
$$

add $\Rightarrow$

$$
\begin{array}{r}
8\ 2\ 8 \\
\times\quad 3\ 5 \\
\hline
4\ 1\ 4\ 0 \\
+\ 2\ 4\ 8\ 4\ 0 \\
\hline
2\ 8\ 9\ 8\ 0
\end{array}
$$

always put a
zero here

This example has shown the steps to explain, but you should be able to do just do the 3rd column once you understand the steps

Example 5 623×235

Way 1

Area Model/Box/Grid Method

	600	**20**	**3**
200	200×600 **120,000**	200×20 **4,000**	200×3 **600**
30	30×600 **18,000**	30×20 **600**	30×3 **90**
5	5×600 **3,000**	5×20 **100**	5×3 **15**

Method:

For each box we FIRST multiply the number on the top of the box with the number on the left of of the box.

We then add all the numbers in the boxes together.

$120,000 + 4,000 + 600 + 18,000 + 600 + 90 + 3,000 + 100 + 15 = 146,405$

Way 3

Long Multiplication (this is just an algorithmic way to do way 1)

do every multiplication with the pink numbers
(carry if we have a two-digit number, just like with addition)

$$
\begin{array}{r}
{}^{1}6\ {}^{1}2\ 3 \\
\times\ 2\ 3\ 5 \\
\hline
3\ 1\ 1\ 5 \\
\end{array}
$$

we write our answers on the top line

next $\Rightarrow$

do every multiplication with the blue numbers
(carry if we have a two-digit number, just like with addition)

$$
\begin{array}{r}
6\ 2\ 3 \\
\times\ 2\ 3\ 5 \\
\hline
3\ 1\ 1\ 5 \\
1\ 8\ 6\ 9\ 0 \\
\end{array}
$$

always put a zero here

we write our answers on the second line

next $\Rightarrow$

do every multiplication with the purple numbers
(carry if we have a two-digit number, just like with addition)

$$
\begin{array}{r}
6\ 2\ 3 \\
\times\ 2\ 3\ 5 \\
\hline
3\ 1\ 1\ 5 \\
1\ 8\ 6\ 9\ 0 \\
+\ 1\ 2\ 4\ 6\ 0\ 0 \\
\hline
1\ 4\ 6\ 4\ 0\ 5 \\
\end{array}
$$

we write our answers on the third line

WE always put two zeros here
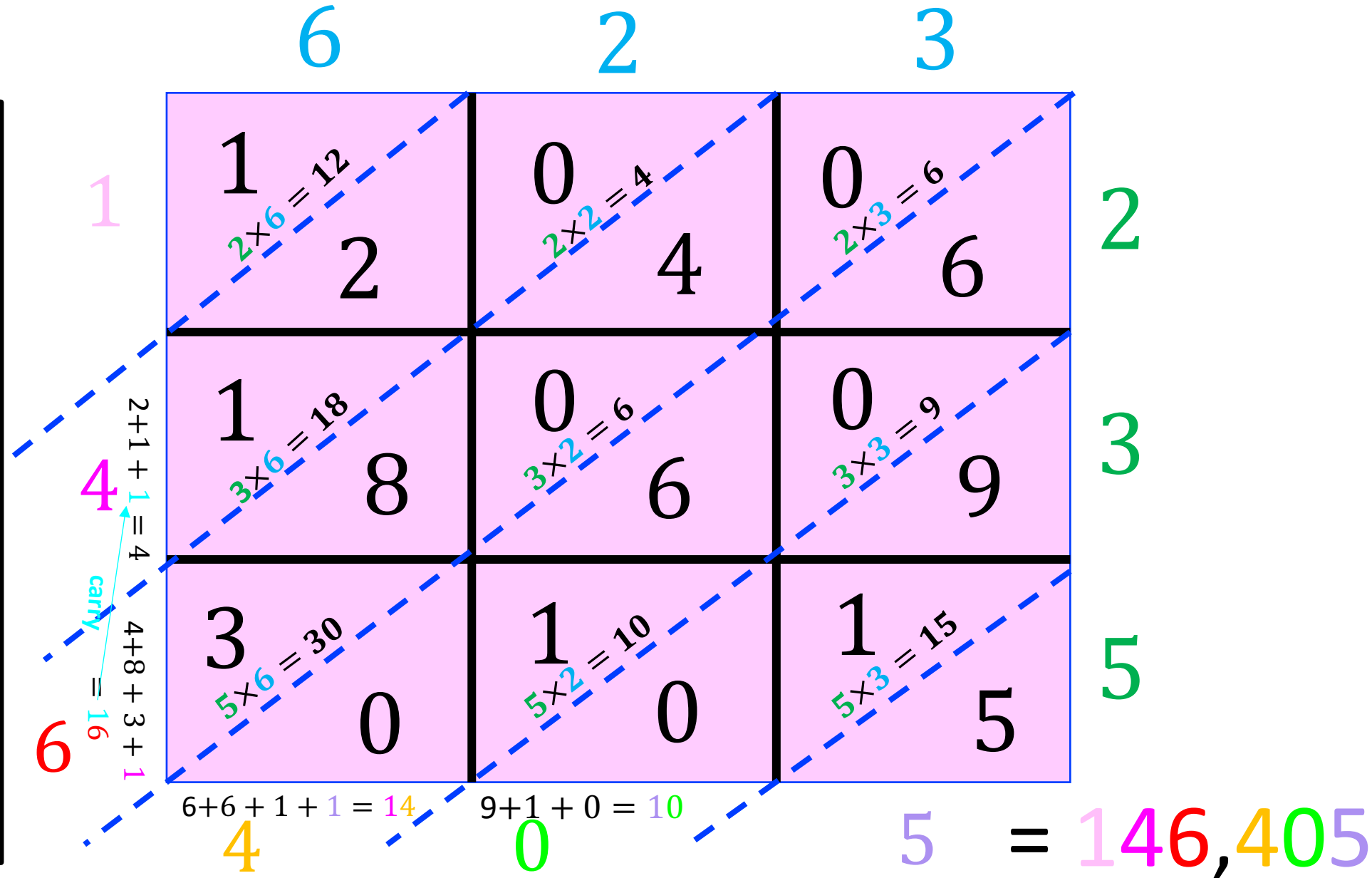
Way 4

Lattice Method

623×235

Method:

Step 1:
For each box we FIRST multiply the numbers on the top of the box with the number on the far right of the box and THEN split the digits of the number you get from multiplying (shown on top of the diagonal) across the dashed diagonal that cuts up each box.

Step 2:
Add the numbers in each of the diagonal strips (start on the right). These numbers form our answer (from left to right).

	6	2	3	
1	1 2 $2\times6=12$	0 4 $2\times2=4$	0 6 $2\times3=6$	2
4	1 8 $3\times6=18$	0 6 $3\times2=6$	0 9 $3\times3=9$	3
6	3 0 $5\times6=30$	1 0 $5\times2=10$	1 5 $5\times3=15$	5

2+1+ 1 = 4   carry   4+8 + 3 + 1 = 16

6+6 + 1 + 1 = 14   9+1 + 0 = 10

4   0   5   = 146,405

Let's now look at ways 5 and 6

Criss Cross Method

and

Chinese Stick Multiplication

23×12

Way 5

Criss Cross Method

$$
\begin{array}{c}
2\ 3 \\
1\ 2 \\
\end{array}
$$

X

③ **do this last**   ② **do this next**   ① **start here**

$2 \times 1 = 2$ $2 \times 2 = 4$ $3 \times 2 = 6$

$1 \times 3 = 3$
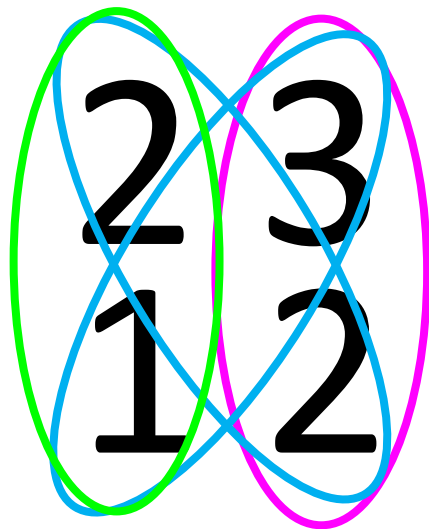
Add these numbers

$4+3=7$

$= 276$

Method:

We multiply each of these combinations

1.

2.

3.
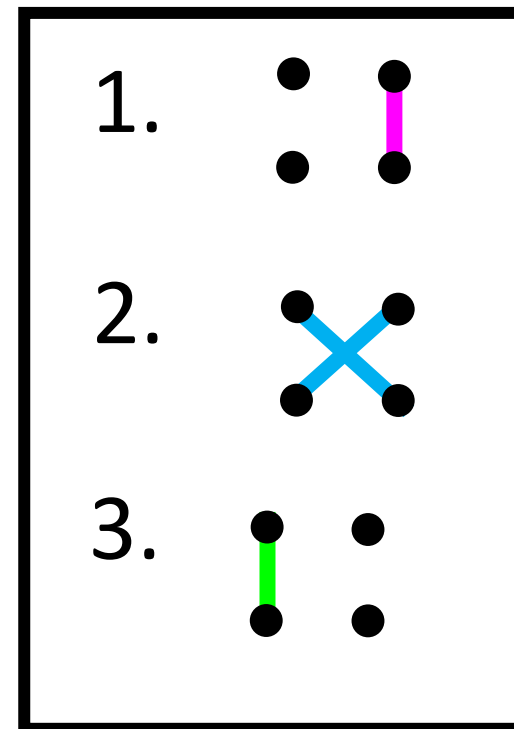
Way 5

Criss Cross Method

$$17 \times 12$$

Method:

We multiply each of these combinations

$$\begin{array}{c} 1\ 7 \\ 1\ 2 \end{array}$$

X
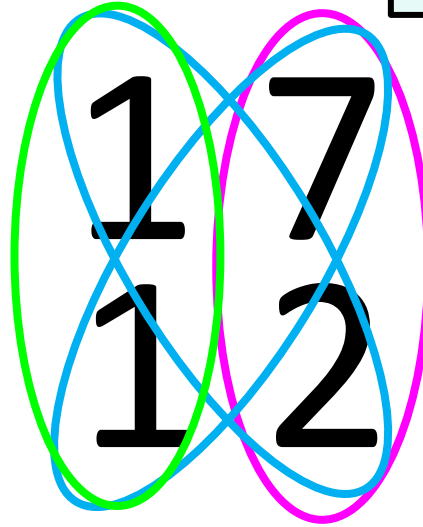
③ **do this last**

$1 \times 1 = 1$

Add

$1+1=2$

② **do this next**

$1 \times 2 = 2$

$1 \times 7 = 7$

Add these numbers

$2+7+1=10$

we carry the **1** to the next sum

① **start here**

$7 \times 2 = 14$

we carry the **1** to the next sum

1.

2.

3.

$$= 204$$
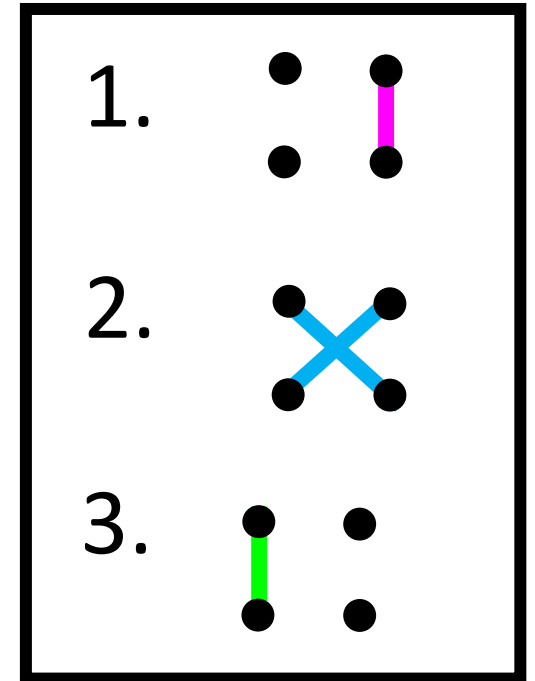
Way 5

Criss Cross Method

78×25

Method:

We multiply each of these combinations

$$\begin{array}{c} 7\ 8 \\ 2\ 5 \end{array}$$

X
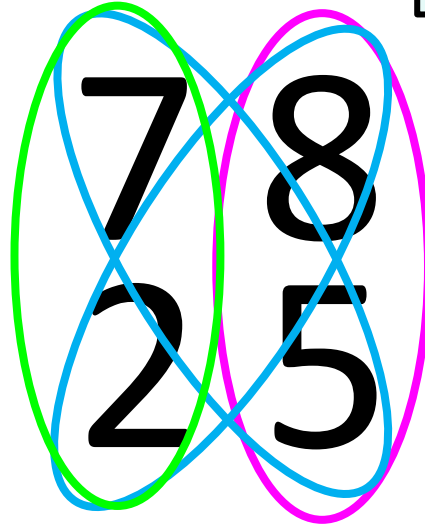
③ **do this last**

$7 \times 2 = 14$

Add

$14 + 5 = 19$

② **do this next**

$7 \times 5 = 35$

$2 \times 8 = 16$

Add these numbers

$35 + 16 + 4 = \textcircled{5}5$
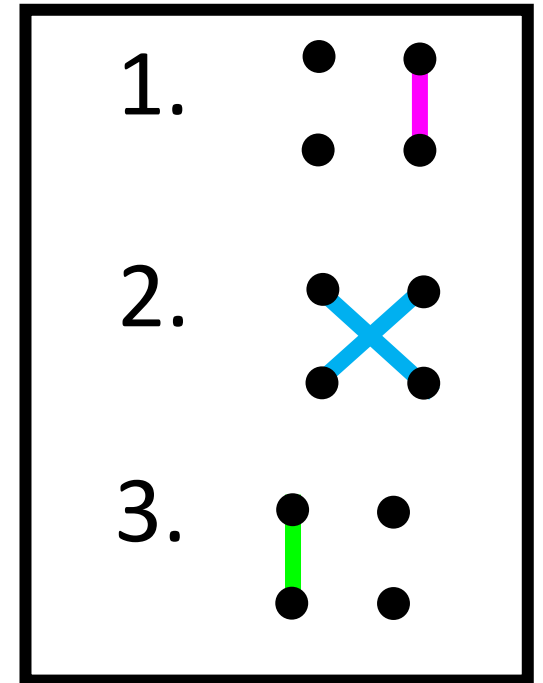
① **start here**

$8 \times 5 = \textcircled{4}0$

we carry the 4 to the next sum

we carry the 5 to the next sum

$$= 1,950$$

Way 5

Criss Cross Method

$$123 \times 231$$

$$1\ 2\ 3$$
$$\times\ 2\ 3\ 1$$
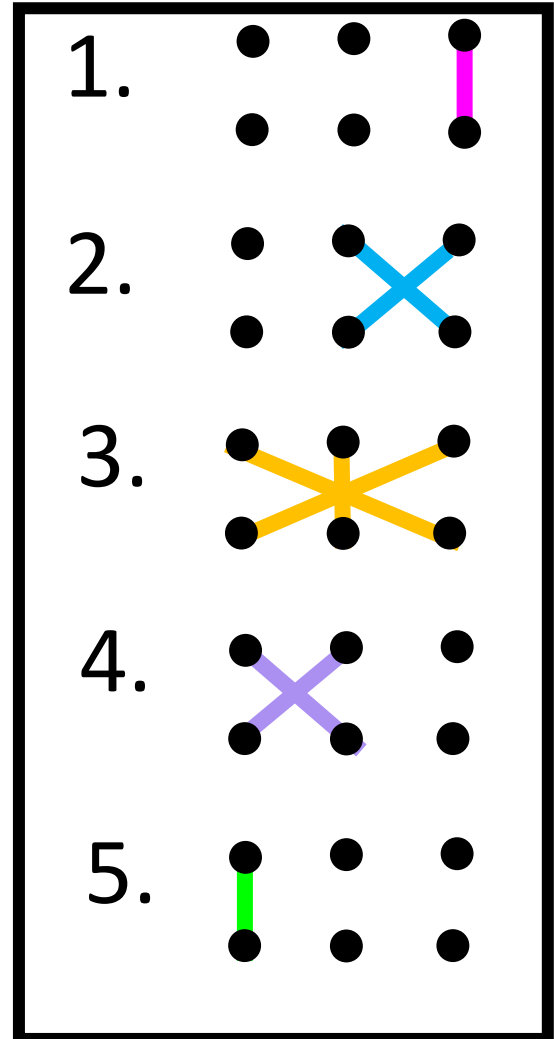
⑤ **do this last**
$1\times2 = 2$

④ **do this fourth**
$1\times3 = 3$
$2\times2 = 4$
Add these numbers
$3+4+1= 8$

③ **do this third**
$2\times3 = 6$
$1\times1 = 1$
$2\times3 = 6$
Add these numbers
$6+1+6+1=14$
we carry the 1 to the next sum

② **do this next**
$2\times1 = 2$
$3\times3 = 9$
Add these numbers
$2+9=11$
we carry the 1 to the next sum

① **start here**
$3\times1 = 3$

we carry the 1 to the next sum

$$= 28,413$$

Method:

We multiply each of these combinations

1.

2.

3.

4.

5.

Way 6

Chinese Stick Multiplication

$$433 \times 12$$

count the intersections for each colour group

$$\mathbf{12} \times \mathbf{433}$$

3 3

3

1

4

2

next $\Rightarrow$

$\mathbf{4} + \mathbf{1}$

move up here and add with the 4

$\textcircled{11}$

9

6

start

$$= \mathbf{5196}$$

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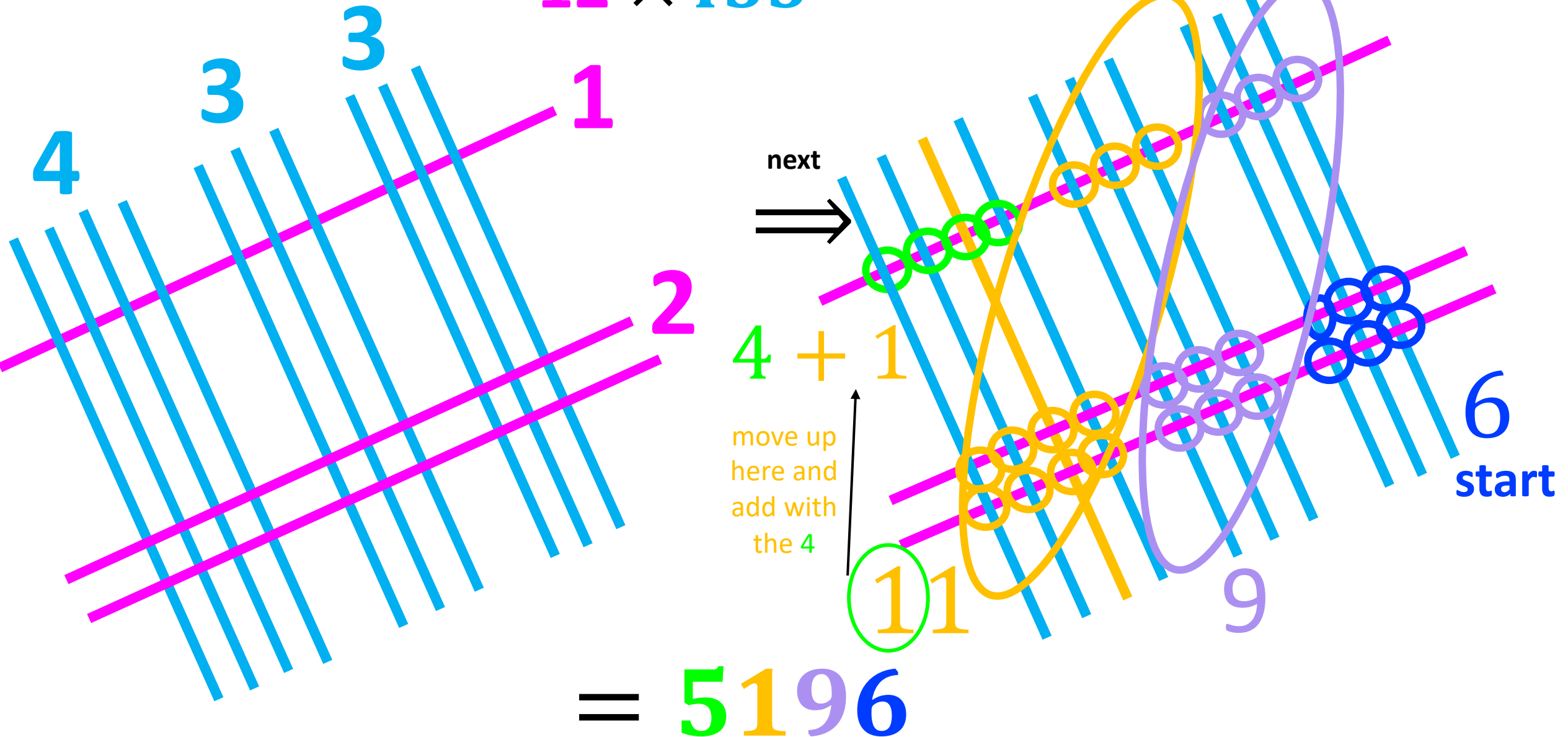
Way 6

Chinese Stick Multiplication 568×976

568×976

$45+10=55$

$89+15=104$
carry the 10

$144+9=153$
carry the 15

$92+4=96$
carry the 9

48 start
carry the 4

$= 554,368$

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